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Bounded Error Approximation Algorithms for Risk-Based Intrusion Response

K Subramani
West Virginia University Research Corporation

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Final Report

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Prior to our work, intrusion response had not been studied within a graph-theoretic framework.
Some of our important contributions include:
(a) The partial vertex cover problem for matchings (PVCM) is poly time solvable, if either the vertices or the edges are weighted, but NP-hard, if both are weighted.

(b) We show that the PVCM admits a fully polynomial approximation scheme, when both vertices and edges are weighted.

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Final Report

K. Subramani
LDCSEE,
West Virginia University,
Morgantown, WV
{K.Subramani@mail.wvu.edu}

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Bounded Error Approximation Algorithms for Risk-Based Intrusion Response

1 Definitions

In this section, we present some definitions that will allow us to formulate our contributions.

Let $G = \langle V, E \rangle$ be a graph, and u is a vertex, e is an edge of G, respectively. We say that u covers e in G, if u is incident to e, that is, u is one of the end-vertices of e. For a set $V' \subseteq V$, let $E_{V'}$ denote the set of edges of G that are covered by vertices of V'.

In the vertex cover (VC) problem, we are given a graph $G = \langle V, E \rangle$, where V is the vertex set with $|V| = n$, E is the edge set with $|E| = m$. The goal is to find a minimum cardinality subset $V' \subset V$, such that $E_{V'} = E$.

In this report we consider the partial vertex cover (PVC) problem restricted to bipartite graphs (PVCB) and its weighted extensions.

Definition 1.1 *Given an integer t, and an undirected bipartite graph $G = \langle V, E \rangle$. The PVCB problem is defined as finding a minimum cardinality subset $S \subset V$ such that $|E_S| \geq t$.*

Definition 1.2 *Given an integer t, an undirected bipartite graph $G = \langle V, E \rangle$ and a weight-function $p : V \to N$. The VPVCB problem is defined as finding a set $S \subset V$ such that $|E_S| \geq t$ and $p(S)$ is minimized.*

Definition 1.3 *Given an integer t, an undirected bipartite graph $G = \langle V, E \rangle$ and an edge-weight function $q : E \to N$. The EPVCB problem is defined as finding a minimum cardinality subset $S \subset V$ such that $q(E_S) \geq t$.*

Definition 1.4 *Given an integer t, an undirected bipartite graph $G = \langle V, E \rangle$ and weight functions $p : V \to N$ and $q : E \to N$. The WPVCB problem is defined as finding a set $S \subset V$ such that $q(E_S) \geq t$ and $p(S)$ is minimized.*

It is trivial to observe that the PVC problem is a generalization of the VC problem. We will also consider the budgeted maximum coverage problem, and its weighted extensions.

Definition 1.5 *Given an integer k, and an undirected bipartite graph $G = \langle V, E \rangle$. The BMCB problem is defined as finding a set $S \subset V$ such that $|S| \leq k$ and $|E_S|$ is maximized.*

Definition 1.6 *Given an integer k, an undirected bipartite graph $G = \langle V, E \rangle$, and a vertex-weight function $p : V \to N$. The VBMCB problem is defined as finding a set $S \subset V$ such that $p(S) \leq k$ and $|E_S|$ is maximized.*

Definition 1.7 *Given an integer k, an undirected bipartite graph $G = \langle V, E\rangle$, and an edge-weight function $q : E \to N$. The EBMCB problem is defined as finding a set $S \subset V$ such that $|S| \leq k$ and $q(E_S)$ is maximized.*

Definition 1.8 *Given an integer k, an undirected bipartite graph $G = \langle V, E\rangle$, and weight functions $p : V \to N$ and $q : E \to N$. The WBMCB problem is defined as finding a set $S \subset V$ such that $p(S) \leq k$ and $q(E_S)$ is maximized.*

For a graph $G = \langle V, E\rangle$ let $\delta(G)$ and $\Delta(G)$ denote the minimum and maximum degree of G, respectively. We say that a set $U \subset V$ is independent in G, if no two vertices of U are joined with an edge. For a positive integer k and a graph G, let $OPT_G(k)$ be the maximum number of edges of G, that can be covered with k vertices of G. We say that a graph G is MNC, if for every $k \geq 1$,

$$OPT_G(k+2) - OPT_G(k+1) \leq OPT_G(k+1) - OPT_G(k).$$

We say that a graph G is weak MNC, if for every $k \geq 1$,

$$OPT_G(k+2) - OPT_G(k+1) \leq 1 + OPT_G(k+1) - OPT_G(k).$$

Definition 1.9 *A graph G is called a matching, if and only if each vertex of G is incident to exactly one edge of G.*

We use the notations WPVCM, VPVCM, EPVCM, and PVCM analogously to denote the Partial Vertex Cover problem and its generalizations on matchings. Finally, we use the notations WPVCP, VPVCP, EPVCP, and PVCP analogously to denote the Partial Vertex Cover problem and its generalizations on paths.

An algorithmic problem is said to be fixed parameter tractable with respect to a parameter k, if there is an algorithm that solves the problem exactly and runs in time $f(k) \cdot n^{O(1)}$. Here f is a function solely depending on k, and n is the size of the input.

2 Our contributions

We were able to come up with several results on partial covers.

(a) We show that PVCM is in **P**. This means that partial vertex cover problem can be solved exactly in polynomial time for matchings.

(b) We show that VPVCM is in **P**. This means that partial vertex cover problem can be solved exactly in polynomial time for vertex-weighted matchings.

(c) We show that EPVCM is in **P**. This means that partial vertex cover problem can be solved exactly in polynomial time for edge-weighted matchings.

(d) We show that WPVCM is **NP-hard**. This means that partial vertex cover problem is unlikely to be solved exactly in polynomial time for vertex-weighted and edge-weighted matchings. On the positive side, we show that there is an FPTAS for the WPVCM problem.

(e) We show that WPVCM is in **P**, if the vertex weights are restricted to $\{1, 2\}$. This means that partial vertex cover problem is can be solved exactly in polynomial time for vertex-weighted and edge-weighted matchings, when vertex-weights are 1 or 2.

(f) We show that WPVCP is **NP-hard**. This means that partial vertex cover problem is unlikely to be solved exactly in polynomial time for vertex-weighted and edge-weighted paths. On the positive side, we show that there is an FPTAS for the WPVCP problem.

(g) We show that VPVCP is in **P**. This means that partial vertex cover problem can be solved exactly in polynomial time for vertex-weighted paths.

(h) We show that EPVCP is in **P**. This means that partial vertex cover problem can be solved exactly in polynomial time for edge-weighted paths.

(i) We show that PVCP is in **P**. This means that partial vertex cover problem can be solved exactly in polynomial time for paths.

(j) We show that all trees are weak MNC. This is important, because it shows that in trees the increase of the value of $OPT_G(k+1) - OPT_G(k)$ can be at most one.

(k) We show that if T is a tree, and $1 \leq L \leq |E(T)|$, then there is an independent set U of vertices of T, such that U covers exactly L edges. The proof of this result is constructive. It implies that the independent set U can be constructed in polynomial time. This has a nice consequence that VPVCB can be solved exactly in polynomial time for degree-weighted trees.

(l) We were able to come up with some classes of graphs for which PVCB and its weighted extensions can be solved exactly in polynomial time, or approximated within a certain factor, that is beats known factors. This classes include bipartite regular graphs, complete k-partite graphs, and the class of graph in which $\delta(G)$ and $\Delta(G)$ are not too far from each other.

(m) We were able to design a linear programming based $\frac{8}{9}$-approximation algorithm for EBMCB. This result implies that BMCB also admits an $\frac{8}{9}$-approximation algorithm. Our result beats the general $\frac{3}{4}$ bound for maximum coverage problem for sets obtained by Ageev and Sviridenko.

(n) We were able to determine the computational complexity of PVCB and its weighted versions, completely, when the input graph is restricted to trees. For this case, PVCB, VPVCB and EPVCB turned out to admit polynomial time algorithms. At the same time, WPVCB remains **NP-hard**. We were able to show that WPVCB admits an FPTAS in this case. The FPTAS is obtained via use of so-called centroid decompositions of trees.

(o) S. Saurabh et. al have shown that PVCB is fixed parameter tractable with respect to k. Here k is the vertex budget. We were able to extend the result of S. Saurabh et. al by proving that EPVCB is also fixed parameter tractable with respect to k. The result implies that EPVCB can be solved exactly in polynomial time if k is small.

Our work has resulted into the papers [1, 2, 3, 4]

3 Future work

The following issues remain unsolved and we plan to address them in future:

(1) Determination of the computational complexity of PVCB and its weighted extensions in bounded degree graphs.

(2) Determination of the computational complexity of PVCB and its weighted extensions in planar graphs.

(3) Design of approximation algorithms for EBMCB whose performance ratio is greater than $\frac{8}{9}$.

(4) Design of approximation algorithms for EPVCB whose performance ratio is smaller than $\frac{4}{3}$.

(5) Investigation of fixed-parameter tractability of VPVCB and WPVCB.

References

[1] Bugra Caskurlu, Vahan Mkrtchyan, Ojas Parekh and K. Subramani. *On Bipartite Graphs, Trees and Their Partial Vertex Covers*. Submitted to ACM Transactions on Algorithms.

[2] Vahan Mkrtchyan, Ojas Parekh, Danny Segev and K. Subramani. *The Approximability of Partial Vertex Covers in Trees*. Submitted to Discrete Optimization.

[3] Bugra Caskurlu, Vahan Mkrtchyan, Ojas Parekh, K. Subramani. *On Partial Vertex Cover and Budgeted Maximum Coverage Problems in Bipartite Graphs*. **Lecture Notes in Computer Science 8705**, (2014), 13-26.

[4] Bugra Caskurlu and K. Subramani. *Partial Vertex Covers of Some Bipartite Graphs*. Unpublished manuscript, (2013).

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Abstract

Our research consisted of modeling the intrusion response problem as one of finding a partial vertex cover in bipartite graphs.
Prior to our work, intrusion response had not been studied within a graph-theoretic framework.
Some of our important contributions include:
(a) The partial vertex cover problem for matchings (PVCM) is poly time solvable, if either the vertices or the edges are weighted, but NP-hard, if both are weighted.

(b) We show that the PVCM admits a fully polynomial approximation scheme, when both vertices and edges are weighted.

(c) We show that the partial vertex cover problem on vertex weighted trees can be solved in polynomial time.

(d) A problem that is closely related to the partial vertex cover problem is the maximum coverage problem. We obtained a new approximation algorithm for this problem.

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(a) Bugra Caskurlu , Vahan Mkrtchyan, Ojas Parekh, K. Subramani.
On Partial Vertex Cover and Budgeted Maximum Coverage
Problems in Bipartite Graphs.
Lecture Notes in Computer Science 8705, 13-26 (2014).

(b) Matthew D. Williamson and K. Subramani.
A new algorithm for the minimum spanning tree verification problem.
Computational Optimization and Applications, 61(1): 189-204 (2015).

(c) K. Subramani, James Worthington.
Feasibility checking in Horn constraint systems through a reduction based approach.
Theoretical Computer. Science 576: 1-17 (2015).

(d) K. Subramani, James Worthington.
On Certifying Instances of Zero-Clairvoyant Scheduling.
Comput. J. 57(1): 129-137 (2014).

(e) Salvatore Ruggieri, Pavlos Eirinakis, K. Subramani, Piotr J. Wojciechowski.
On the complexity of quantified linear systems. Theor. Comput. Sci. 518: 128-134 (2014).

(f) R. Chandrasekaran, K. Subramani.
A combinatorial algorithm for Horn programs. Discrete Optimization 10(2): 85-101 (2013).

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